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EFFECT OF GAS BUBBLES ON LIQUID METAL HEAT TRANSFER

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Abstract—A theory is developed for heat transfer to liquid metals when the heat transfer surface is covered with gas bubbles. The system analysed is that of constant heat flux to slug flow in a circular pipe. The effective Nusselt number depends upon the thickness of the layer of bubbles and its thermal conductivity. A method of calculating these quantities for systems where the contact angle is less than about 60° is described. A number of previously unexplained experimental results can be understood in terms of this theory.

NOMENCLATURE

a,	radius of the bubble free core of the flow;
b,	internal radius of tube;
c,	specific heat of liquid metal;
D,	$a^2+(1-\alpha)(b^2-a^2)$;
$F(\theta)$,	$\sin\theta_0(\cos\theta_a-\cos\theta_r)$;
h,	heat transfer coefficient;
K,	thermal conductivity of liquid metal;
K_1,	effective thermal conductivity of bubble layer;
Nu,	Nusselt number in absence of gas bubbles, $h2b/K$;
Nu_{eff},	Nusselt number with gas bubbles on surface;
Pe,	Peclet number, $u2b\rho c/K$;
Q,	heat input/unit length;
r,	radius;
t,	thickness of bubble layer;
T,	temperature;
T_m,	mean temperature of bubble layer flow ($a<r<b$);
T_0,	mean temperature of whole flow;
T_0^1,	mean temperature of core flow ($r<a$);
u,	mean flow velocity;
α,	void fraction in the bubble zone;
$\theta_0, \theta_a, \theta_r$,	equilibrium, advancing and receding contact angles, $\theta_0=(\theta_a+\theta_r)/2$;
ρ,	density;
τ,	dimensionless bubble layer thickness, $t/2b$.

INTRODUCTION

POOR heat transfer to liquid metals has frequently been attributed to the presence of entrained gas [1–4]. Whether the gas bubbles are dispersed in the liquid metal and flow with it, or they adhere to the heat transfer surface, is not clear. If the gas is in the form of spherical bubbles dispersed in the flow it can significantly affect the heat transfer only if the void fraction is greater than about 10^{-2}. There is no evidence to support the suggestion that the gas can exist in the form of thin sheets [1], which would of course have a much greater effect on the thermal conductivity of the mixture. In this paper it is assumed that the gas bubbles adhere to the surface, and that the void fraction of the flowing liquid is small. This layer of bubbles on the surface could arise either from bubbles in the flow hitting the surface and sticking, or from dissolved gas coming out of solution.

THEORY

The system analysed is that of constant heat flux to liquid metal flowing in a circular pipe. A slug flow model is adopted, i.e. the velocity is the same at all points within the pipe and heat transfer is by conduction. Axial conduction is neglected and constant fluid properties assumed. A layer of thickness t by the wall contains the bubbles, as shown in Fig. 1. This layer

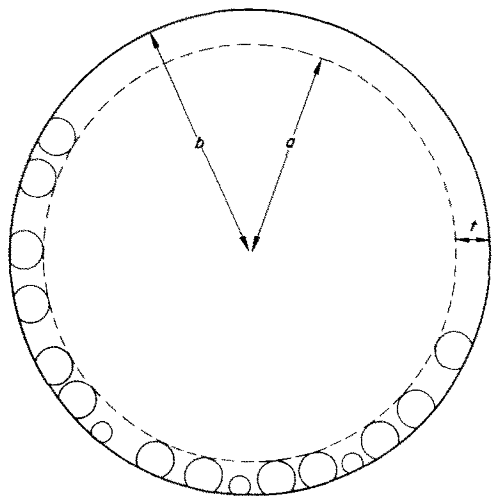

FIG. 1. The walls of the pipe are assumed covered with a layer of gas bubbles, thickness t.

will be of uniform thickness except possibly in horizontal tubes with low flow rates when it may be distorted by buoyancy forces. The properties of this layer are determined by the void fraction, α, assumed constant across the layer, and effective conductivity K_1. It is assumed that any bubbles that grow beyond this layer are swept off into the flow, i.e. t is itself a function of the flow velocity. In the core of the flow ($r < a$) the void fraction is effectively zero and the properties are those of the pure metal.

Once the flow is fully developed thermally the rate of temperature rise is the same at all radial positions, so the heat being conducted through a cylindrical surface radius r must be in proportion to the thermal capacity of the material within that surface.

So if the total heat flowing into unit length of the pipe is Q, the heat flux at $r = a$ is

$$\frac{Q}{2\pi a}\left\{\frac{a^2}{a^2+(1-\alpha)(b^2-a^2)}\right\}$$

and hence

$$T_a - T_0^1 = Qa^2/NuK\pi D \quad (1)$$

where Nu is the Nusselt number of the core flow, T_0^1 the mixed mean temperature of the core and

$$D = a^2+(1-\alpha)(b^2-a^2).$$

Similarly the total heat through a cylindrical surface radius $r(>a)$ is

$$Q\left\{\frac{a^2+(1-\alpha)(r^2-a^2)}{D}\right\} = K_1 2\pi r\frac{\mathrm{d}T}{\mathrm{d}r}$$

giving

$$T_r = T_b - \frac{Q}{2\pi K_1 D}\{\alpha a^2 \log b/r + (1-\alpha)(b^2-r^2)/2\} \quad (2)$$

and hence

$$T_a = T_b - \frac{Q}{2\pi K_1 D}\{\alpha a^2 \log b/a + (1-\alpha)(b^2-a^2)/2\}. \quad (3)$$

The mean value of the temperature of the liquid between a and b is given by integrating (2) between these limits and is

$$T_m = T_b - \frac{Q}{2\pi K_1 D}\left[\alpha a^2 \log b + (1-\alpha)(b^2-a^2)/4 + \alpha a^2/2 - \alpha a^2\frac{b^2\log b - a^2\log a}{b^2-a^2}\right]. \quad (4)$$

The mean temperature of the whole flow, T_0, is related to T_0^1 and T_m by

$$T_0 D = T_0^1 a^2 + T_m(1-\alpha)(b^2-a^2) \quad (5)$$

eliminating T_a, T_0^1 and T_m from (1), (3), (4) and (5) gives, with some simplification,

$$\frac{(T_b-T_0)D^2\pi}{Q} = \frac{a^4}{KNu} + \frac{1}{2K_1}\{\alpha^2 a^4\log b/a + (1-\alpha^2)a^2(b^2-a^2)/2 + (1-\alpha)^2(b^2-a^2)^2/4\}$$

hence the effective Nusselt number, i.e. the value that would be measured using the usual definition,

$$Nu_{\text{eff}} = \frac{Q}{2\pi b(T_b-T_0)}\frac{2b}{K}$$

is given by

$$Nu_{\text{eff}} = \frac{\{1-\alpha+\alpha(a/b)^2\}^2 Nu}{(a/b)^4 + KNu\{\alpha^2(a/b)^4\log b/a + (1-\alpha^2)(a/b)^2(1-(a/b)^2)/2 + (1-\alpha)^2(1-(a/b)^2)^2/4\}/2K_1}$$

in terms of the dimensionless bubble layer thickness $\tau = t/2b$ this becomes

$$\frac{Nu_{\text{eff}}}{Nu} = \frac{\{1-4\alpha\tau(1-\tau)\}^2}{\{1-2\tau\}^4} \times \frac{1}{1+KNu\{2(1-\alpha)^2\tau^2(1-\tau)^2(1-2\tau)^{-4} + (1-\alpha^2)\tau(1-\tau)(1-2\tau)^{-2} - (\alpha^2/2)\log(1-2\tau)\}/K_1}. \quad (6)$$

In a fully consistent slug flow model Nu in the above expression would be 8. However, if we regard Nu as the Lyon–Martinelli value [5] in the absence of gas bubble effects, i.e.

$$Nu = 7 + 0{\cdot}025\,Pe^{0{\cdot}8}$$

then we have the obviously correct result $Nu_{\text{eff}} = Nu$ when $\tau = 0$. In practice τ only becomes appreciable for low Peclet numbers, when the Lyon–Martinelli value of the Nusselt number is close to the slug flow value of 8 anyway. So we will take Nu to be the Lyon–Martinelli value.

EVALUATION OF Nu_{eff}

In order to evaluate (6) we need to know α, τ and K_1. Estimates of τ and K_1 are both possible provided the gas bubbles can be treated as spheres, i.e. the contact angle with the surface is not too large. For example, if the condition is imposed that the volume of the gas bubble on the surface must be at least 80 per cent that of a complete sphere of the same radius the equilibrium contact angle θ_0 must be less than 65°. So one might reasonably expect the approach to be described in this section to be valid for systems where the liquid wets the surface ($\theta_0 < 90°$), but not for non-wetting systems ($\theta_0 > 90°$).

The effective thermal conductivity K_1 of a medium of conductivity K containing spheres of zero conductivity is given by the Maxwell equation [6]

$$K_1 = K(1-\alpha)/(1+\alpha/2). \qquad (7)$$

The thickness of the bubble layer τ is determined by the maximum size the bubbles can grow to before being swept off into the flowing liquid. The critical size depends on the drag, surface tension and buoyancy forces acting on the bubble. Methods of calculating these forces are described in reference [7]. The drag force is found using expressions for the drag coefficient determined in experiments on freely rising bubbles, so again a small contact angle giving essentially a complete sphere is required. The experimental results show that this is a good approximation for $\theta_0 = 50°$ [7]. The surface tension force results from the different contact angles upstream and downstream of the bubble, and depends on the parameter.

$$F(\theta) = \sin\theta_0(\cos\theta_r - \cos\theta_a)$$

where θ_0 is the equilibrium contact angle, θ_a and θ_r the limiting advancing and receding contact angles upstream and downstream of the bubble. If $\theta_0 < 65°$ then the maximum value of $F(\theta)$ for the theory to be valid is around 0·6 (assuming $\theta_a - \theta_r$ values of up to 40°).

The remaining quantity is α, the void fraction of the bubble layer. This could take a large range of values, but it seems likely that in some circumstances the process whereby the bubbles appear on the wall could be so effective that the void fraction would be determined by how close the bubbles could be packed together. This provides an upper limit to the reduction in Nusselt number that could be expected. Hexagonal close packing of spheres on a surface would give a void fraction of 0·6, while close packing on a square lattice would give a value of just over 0·5. It is not considered that either of these values could be realised in practice, bubbles in contact would coalesce, become larger than the critical size and be swept off the surface. So we will take $\alpha = 0{\cdot}4$ as an upper limit.

As an illustration of the reduction in Nusselt number that is possible equation (6) is plotted in Fig. 2 as a function of the dimensionless bubble layer thickness τ for $\alpha = 0{\cdot}4$, $Nu = 8{\cdot}4$, and using equation (7). For values of τ greater than about 0·1 Nu_{eff}/Nu reaches a value of about 0·5.

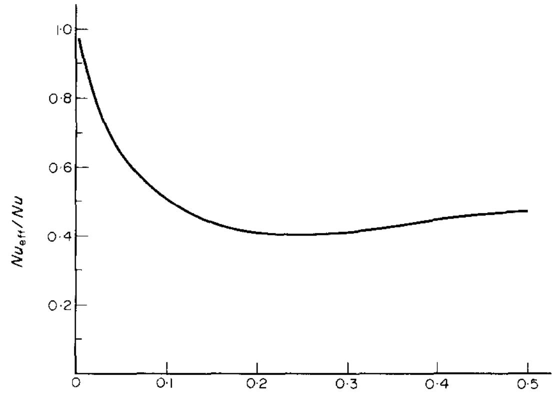

FIG. 2. Effective Nusselt number as a function of dimensionless bubble layer thickness τ. From equation (6) with $\alpha = 0{\cdot}4$.

Using the methods described in [7] values of τ for sodium at 400°C in a 15 mm i.d. horizontal pipe were calculated for various values of the surface tension parameter $F(\theta)$, giving the values of Nu_{eff} shown in Fig. 3. For the smaller bubble radii the equations in [7] could be used directly, for larger bubbles the drag coefficient changes (above a critical value of the bubble Reynolds number), and the appropriate expression for the drag coefficient from [8] was used. Figure 3 shows that the reduction in the heat transfer performance becomes serious at low values of the Peclet number,

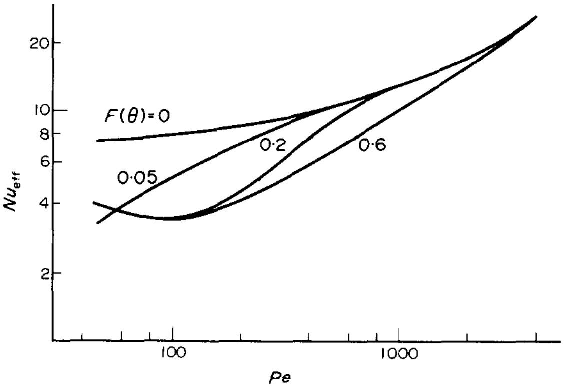

FIG. 3. Calculated values of effective Nusselt number for sodium in a 15 mm i.d. horizontal pipe, for various values of the contact angle parameter $F(\theta)$. $\alpha = 0{\cdot}4$.

and below $Pe = 100$ the 50 per cent reduction in Nusselt number is achieved for a wide range of possible values of $F(\theta)$. Use of the theory at much lower values of Pe is probably not justified since the diameter of the bubbles is becoming comparable with the radius of the tube.

COMPARISON WITH EXPERIMENTAL RESULTS

A search of the literature revealed only one experiment where the Nusselt number was measured for constant heat flux to a wetting liquid metal in a circular pipe, and where the authors considered the results were affected by entrained gas. This was the work of MacDonald and Quittenton [1, 9] who studied heat transfer to sodium flowing down a 15·9 mm i.d. vertical pipe. The measurements covered the Peclet number range of 100–220, the great majority of the data being in the range 120–200. The values of the Nusselt number found ranged from slightly over the Lyon–Martinelli value to about 50 per cent of it. A glance at Fig. 3 suggests the present theory could account for this variation with suitable values of $F(\theta)$.

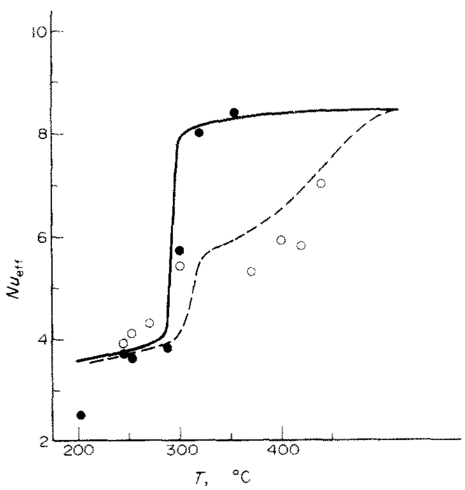

FIG. 4. Experimental results of [9], open circles represent measurements with increasing temperature, solid circles those with decreasing temperature. The dashed line is the prediction of equation (6) for a high estimate of $F(\theta)$, the solid line that for a low estimate.

The most striking feature of the experimental results was the marked improvement in heat transfer with increasing temperature. The experimental points are shown in Fig. 4. As in [1] the mean Nusselt number at each temperature has been calculated, but because of the marked hysteresis in the data a distinction has been made between data obtained with increasing temperature and that with decreasing temperature. That is, an individual Nusselt number measurement went into one category if the test section had previously always been at a lower temperature, and into the other if the test section had at any time been at a higher temperature.

A comparison with the theory is difficult because of the unknown values of $F(\theta)$. However it is possible to estimate reasonable limits. The test section was made of Monel (R), a nickel alloy with 30 per cent copper. The author is not aware of any contact angle data for the alloy, nor for copper, beyond the observation [10] that the wetting behaviour of copper is similar to that of nickel. It is assumed therefore that data for nickel can be used.

It is generally agreed that for nickel as for many other metals there is a critical wetting temperature with sodium of around 200°C [11], below this temperature the contact angles are large (>90°) and so the assumption of spherical gas bubbles on the surface is invalid. Above this temperature the contact angles rapidly become quite small. Addison and co-workers [12] found a critical wetting temperature for nickel of 195°C, and at 200°C the receding contact angle became zero after about half an hour. Longson and Thorley [13] similarly found a receding contact angle of zero at temperatures above 200°C, but it took much longer to reach this value, 3 days at 215°C, 1 h at 270°C. Also finite contact angles have been reported right up to 500°C [14]. Less work appears to have been done on advancing contact angles but finite values up to 250°C (the highest temperature investigated) have been observed [10].

So the method of calculation outlined earlier for bubbles that can be regarded as spherical should become valid at some temperature slightly above 195°C. 195 and 220°C are taken as limits ($F(\theta) = 0{\cdot}6$ here). By 300°C it seems likely that $\theta_r = 0$ with say $\theta_a = 20°$. However as an upper limit to the surface tension force $\theta_r = 10$ and $\theta_a = 40°$ are also considered. At some point θ_a will itself become zero, perhaps at 400 or 500°C. These limiting values can be considered to give upper and lower limits to $F(\theta)$ throughout the range, with linear interpolation between the reference points.

In this way the two theoretical lines on Fig. 4 are obtained, for a mean Peclet number of 160. Since in this case the test section is vertical the buoyancy force on the bubbles has also been included in calculating the bubble radius on departure. Clearly the experimental results are consistent with the theory, and even better agreement could be obtained for a slightly different variation of $F(\theta)$ with temperature. Also the hysteresis in the results is a natural consequence of the influence of the surface tension force on the thickness of the bubble layer. The good wetting achieved at the high temperature is retained to some extent as the temperature is lowered.

The experimental Nusselt number of 2·5 found at 206°C is below the value predicted by the theory, but as already explained the theory will break down at some temperature in this region. It is likely that this low value represents the onset of non-wetting conditions. Since in the critical region close to the surface the void fraction is likely to be higher and the effective thermal conductivity lower for non-wetting conditions ($\theta > 90°$) a lower value of Nu is to be expected.

The theory explains both the magnitude and the temperature dependence of the effects observed by MacDonald and Quittenton. It appears therefore that the walls of their test section must have been covered with a dense layer of adhering bubbles. Since the test section was heated, and the solubility of the argon cover gas used increases with temperature, this layer of bubbles cannot have been due to dissolved gas coming out of solution. Presumably gas bubbles entrained in the flow striking the surface stuck to it. The fact that this does not usually happen in air–water systems may simply be due to the almost universal surface contamination in such systems.

The explanation advanced by MacDonald and Quittenton for the temperature dependence, that the gas bubbles in the flow dissolve at higher temperature, is not consistent with the known solubility of argon in sodium [15], nor with the very short time available between the expansion tank, where the gas was presumably entrained, and the test section.

Although the theory strictly applies to constant heat flux and circular pipes it is interesting to examine the experimental literature on heat transfer to liquid metals more generally, including other boundary conditions and geometries. Three cases were found where the liquid metal was cooled rather than heated, with the possibility of dissolved gas coming out of solution on the heat transfer surface. All three of these showed reduced Nusselt numbers. In one case [2] the authors considered that part of the test section might have been completely covered in gas due to a fall in the level of the liquid metal, so any effect due to dissolved gas coming out of solution would have been masked. In the second experiment [16] sodium–potassium alloy (NaK) flowing down a circular tube was cooled in a shell and tube heat exchanger. The Nusselt number was measured over a wide range of Peclet number, being in agreement with theory at $Pe = 2500$, but some 20 per cent lower by $Pe = 250$. Tidball [17] measured shell side heat transfer coefficients in two identical shell and tube heat exchangers, one heated by NaK and the other cooled by NaK. The heat transfer coefficient for the cooled flow was 20 per cent less than that for the heated one.

CONCLUSIONS

The theory developed in this paper predicts reductions in Nusselt number for liquid metals under wetting conditions of up to 50 per cent at low Peclet numbers. This reduction is due to a layer of gas bubbles adhering to the surface. At high Peclet numbers, i.e. high flow rates, only very small gas bubbles can withstand the drag forces, and the effect on heat transfer is negligible.

Experimental results described in the literature suggest that this effect has already been observed, and the layer of gas bubbles can build up either by entrained gas striking the surface and sticking to it, or by dissolved gas coming out of solution.

Similar but larger reductions in heat transfer are to be expected under non-wetting conditions.

REFERENCES

1. W. C. MacDonald and R. C. Quittenton, Critical analysis of metal wetting and gas entrainment in heat transfer to molten metal, *Chem. Engng Prog. Symp. Ser.* **50**(9), 59–67 (1954).
2. V. I. Subbotin, F. A. Kozlov and N. N. Ivanovskii, Heat transfer to sodium under conditions of free and forced convection and when oxides are deposited on the transfer surface, *High Temperature* **1**, 368–372 (1963).
3. T. Mizushina *et al.*, Effect of gas entrainment on liquid metal heat transfer, *Int. J. Heat Mass Transfer* **7**, 1419–1425 (1964).
4. C. L. Rickard, O. E. Dwyer and D. Dropkin, Heat transfer rates to cross-flowing mercury in a staggered tube bank, *Trans. Am. Soc. Mech. Engrs* **80**, 646–652 (1958).
5. R. N. Lyon, Liquid metal heat transfer coefficients, *Chem. Engng Prog. Symp. Ser.* **47**(2), 75–79 (1951).
6. M. Jacob, *Heat Transfer*, Vol. 1, p. 83. John Wiley, N.Y. (1949).
7. R. H. S. Winterton, Sizes of bubbles produced by dissolved gas coming out of solution on the walls of pipes in flowing systems, *Chem. Engng Sci.* **27**, 1223–1230 (1972).
8. F. N. Peebles and H. J. Garber, Studies on the motion of gas bubbles in liquids, *Chem. Engng Prog. Symp. Ser.* **49**(2), 88–97 (1953).
9. R. C. Quittenton, The direct measurement of the film coefficient of heat transfer to molten sodium metal in forced convection, Ph.D. Thesis, Univ. of Toronto (1953).
10. D. O. Jordan and J. E. Lane, Wetting of solid metals by liquid alkali metals, Chem. Soc. (Lond.) Spec. Publ. No. 22 147–152 (1967).
11. C. C. Addison, E. Iberson and R. J. Pulham, Influence of surface films on wetting of transition metals by sodium, Soc. Chem. Ind. Monograph No. 28 Surface Phenomena of Metals (1968).
12. C. C. Addision, E. Iberson and J. A. Manning, Liquid metals Part V. The role of oxide films in the wetting of iron, cobalt and nickel by liquid sodium, *J. Chem. Soc.* 2699–2705 (1962).
13. B. Longson and A. W. Thorley, The wetting behaviour of some alloys based on iron, nickel and chromium in liquid sodium, Chem. Soc. Spec. Publ. No. 22 p. 153 (1967).
14. Z. A. Kazakevich and E. A. Zhemchuzhina, *Isv. Vyssh. Ucheb. Zaved., Tsvet. Met.* **10**, 38–40 (1967).
15. K. Thormeier, Solubility of noble gases in liquid sodium, *Nucl. Engng Design* **14**, 69–82 (1970).
16. R. A. Baker and A. Sesonske, Heat transfer in sodium potassium alloy, *Nucl. Sci. Engng* **13**, 283 (1962).
17. R. A. Tidball, Performance of small liquid metal heat exchangers, *Chem. Engng Prog. Symp. Ser.*, No. 5, 43–49 (1953).

EFFET DES BULLES GAZEUSES SUR LE TRANSFERT THERMIQUE PAR METAL LIQUIDE

Résumé—On développe une théorie sur le transfert thermique par métal liquide lorsque la surface de transfert est recouverte de bulles gazeuses. Le système analysé est celui d'une densité de flux constante et d'un écoulement en bloc dans un tuyau circulaire. Le nombre de Nusselt effectif dépend de l'épaisseur de la couche de bulles et de sa conductivité thermique. On décrit une méthode de calcul de ces quantités pour des systèmes où l'angle de contact est inférieur à 60°. Cette théorie permet de comprendre certains résultats expérimentaux jusqu'ici inexpliqués.

DER EINFLUß VON GASBLASEN AUF DEN WÄRMEÜBERGANG BEI FLÜSSIGEN METALLEN

Zusammenfassung—Es wird eine Theorie entwickelt, für den Fall daß beim Wärmeübergang an flüssige Metalle die wärmeabgebende Oberfläche mit Gasblasen bedeckt ist. Im untersuchten System, einem kreisrunden Rohr, herrscht konstante Wärmestromdichte und Propfenströmung. Die wirkliche Nusselt-Zahl hängt ab von der Dicke der Blasenschicht und ihrer Wärmeleitfähigkeit. Für Systeme mit Benetzungswinkel die kleiner als etwa 60° sind wird eine Methode angegeben um beide Größen zu berechnen. Eine große Anzahl früherer ungeklärter Ergebnisse wird mit Hilfe dieser Theorie verständlich.

ВЛИЯНИЕ ПУЗЫРЬКОВ ГАЗА НА ТЕПЛООБМЕН ЖИДКИХ МЕТАЛЛОВ

Аннотация — Разработана теория переноса тепла к жидким металлам от греющей поверхности, покрытой пузырьками газа. В основу расчёта положена схема ползущего течения в круглой трубе при постоянном тепловом потоке на стенке. Эффективное число Нуссельта зависит от толщины и теплопроводности слоя пузырьков. Предлагается метод расчёта указанных величин в случае, когда угол контакта меньше $\sim 60°$. На этой основе можно объяснить некоторые экспериментальные результаты, ранее не поддававшиеся объяснению.